37p.

N62-16514

NASA TN D-1486

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TECHNICAL NOTE

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THE LOCATION AND SIMULATED REPAIR OF ROUGH AREAS OF A GIVEN RUNWAY BY AN ANALYTICAL METHOD

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

WASHINGTON October 1962

N 62-16514

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SUMMARY

This paper presents a method by which specific runways were studied by means of the time histories of the response of a simplified simulation of an airplane taxiing along the simulated runways at various speeds. The actual runway profile data which were obtained by surveys at 2-foot intervals were converted to analog signals and recorded on magnetic tape in the form of FM (frequency modulated) signals proportional to the runway profile. The FM magnetic tape proved to be a convenient means of providing a repeatable analog signal to represent the runway input to a simulated airplane.

An evaluation of the simulated airplane response to runway roughness by means of time histories appears to be an appropriate means of locating relatively rough areas of a given runway. A detailed study of each rough area may be made by this method to determine the minimum repairs necessary to reduce the airplane response to an acceptable level. The time histories of the airplane response while taxiing on two runways at speeds from 50 to 160 knots were compared and the response due to one runway was found to be much greater than the other for most of the speed range even though the power spectra were about the same for each runway.

INTRODUCTION

The problem of runway roughness has been the subject of research and investigation for several years. From the standpoint of runway construction and maintenance the problem has been that of measuring and defining roughness as acceptable or unacceptable. Much work has been done in this area by comparing the power spectra of various runway profiles which are known from experience to be smooth or rough. (For example, see refs. 1, 2, and 3.) Based on this work some standards, which specify certain allowable deviations in elevation for a given distance, have been proposed such as in references 2 and 3. From the standpoint of aircraft manufacturers and operators, the problem has been studied as to the loads imposed on the airplane while taxiing on the runways. (See refs. 4

and 5.) Recently, the aircraft operators have also become concerned with the effects of runway roughness on the take-off performance of jet transports. Some pilots have reported that the airplane response to runway roughness at higher speeds seriously affects their ability to rotate properly during the take-off.

When airport operators receive such reports, the problem becomes a specific one and much of the previously mentioned research is of a too general nature to offer a solution to the problem. Although a power spectrum of the runway profile might indicate that the runway in question is rough, it will not indicate exactly which areas are bad. As a matter of economy, since runway construction or repair is extremely costly, it is necessary to know just which areas need repair. The nature of this problem suggests an approach making use of time histories rather than statistics or power spectra.

This paper presents a method by which a specific runway may be studied by means of the time histories of the response of a simplified simulation of an airplane taxiing along the runway at various speeds. The actual runway profile data, which have been obtained by surveys at 2-foot intervals, were used as the input and the response to this input at various horizontal speeds was computed for a simulated airplane. The simulated airplane represented a modern turbojet transport having rigid body pitch and translation modes, no flexible modes being considered. The landing gear was represented by a bicycle-type arrangement with linear characteristics. After studying the airplane response histories to the runway input, certain areas of the runway were determined to be in need of repair. The runways were then "repaired" by leveling certain holes and bumps and the resulting time histories of airplane response are given.

This procedure may utilize either digital or analog computation of response; however, in this study analog methods were used. The runway profile inputs were obtained from an FM (frequency modulated) tape. The method used to produce the analog runway profile tapes from the digital runway survey data is believed to be unique and is described in the appendix.

SYMBOLS

C_d shock strut damping factor, lb-sec/ft

k spring constant, lb/ft

m mass, lb-sec^2/ft

r radius of gyration of upper mass, ft

ζ damping ratio

ω frequency, radians/sec

Φ power spectral density, $\frac{ft^2}{radians/ft}$

θ angle of pitch, radians

λ wave length, ft

z vertical displacement, ft

Subscripts:

1 lower mass; wheel, tire, axle, and movable portion of strut

2 upper mass; fuselage, wings, etc.

n nose gear

m main gear

T total

t tire

s shock strut

cg center of gravity

R runway

p pilot's compartment

Dots over symbols denote derivatives with respect to time.

PROCEDURE

This investigation utilized the Langley Research Center electronic analog computing facility and tape playback and demodulator equipment. Specific runways were studied by means of the response histories of a simplified simulation of an airplane taxiing along the runway at various constant speeds from 50 to 160 knots. The runway profile input to the simulated airplane was in the form of an analog signal recorded on an FM tape. As a matter of convenience, the tape playback speeds were held constant at either 15 or 30 inches per second and the analog computer time constants were adjusted for each run to represent a given constant airplane speed along the runway.

Runway Simulation

The runway profiles used in this study were obtained by surveying sections of active runways from two large commercial airports. The elevation measurements were obtained by use of level and rod at 2-foot intervals along the length of the runway for a distance of 4,200 feet for runways A and B and 4,000 feet for runway C. This digital data was then converted to an analog form and recorded as an FM signal on magnetic tape. For each runway section two data channels were required, one to represent the runway input to the nose gear of the airplane and one channel with the same data displaced 50 feet to the rear to represent the same runway input to the main gear. For identification purposes, 100-foot runway distance marks were recorded on a third channel. A detailed description of the digital to analog conversion is given in the appendix.

Airplane Simulation

The airplane simulated in this investigation represents a modern jet transport as nearly as possible while keeping a simple mathematical representation. The airplane representation is given in figure 1 along with its physical characteristics. It is realized that an actual aircraft landing gear is an extremely nonlinear device and cannot be easily approximated by linearization as far as realizing the proper response amplitudes. However, the simulated airplane is believed to represent a typical jet transport in that the resonant frequencies of the rigid body pitch and translation modes are representative. As mentioned in reference 2, these lower frequency modes are of primary interest in the response to runway roughness. The frequency-response characteristics of the simulated airplane are given in figure 2. Although the amplitude response is questionable as far as giving the correct magnitude of acceleration response to a given input, it is thought that the relative response to runway roughness will indicate which sections of runway cause the most severe response. Correspondingly, these same areas should cause the most severe response of an actual airplane which has these same resonant frequencies in pitch and translation. The bicycle-gear arrangement called for the main gear to follow the same profile as the nose gear. It can be assumed that the single main gear represents two main landing gear traveling over a profile which is constant in the lateral direction. The increased complexity required to simulate a tricycle landing gear would not be warranted for the purpose of determining the location and amount of runway repairs required.

Assumptions

A level of airplane acceleration response to runway roughness has not been established yet which will determine whether a section of runway is satisfactory or in need of repair. It will then be assumed that the acceleration response of airplanes operating on the runways investigated here has been found to be objectionable and that it is necessary to locate the rough areas which cause this objectionable airplane response. In addition, it will also be assumed that simulator values of acceleration at either the pilot's compartment or the airplane center of gravity which are greater than $\pm 0.5g$ have been established as being undesirable.

RESULTS AND DISCUSSION

The Runway Analog

One significant feature of this investigation was the digital-to-analog conversion of the runway profile data. (See appendix.) The resulting runway analog recorded on magnetic tape was in the form of an FM signal which, after being demodulated, produced a continuously varying voltage that was proportional to the runway elevation. This runway tape provided a convenient means of generating a repeatable analog representation of a given runway. Figure 3 shows the analog representation of runway A. The overall accuracy of the tape, playback, and demodulators was measured and found to be within ± 0.01 foot. Since the original runway survey data were read accurately to 0.01 foot and estimated to 0.001 foot, the digital-to-analog conversion was accomplished with no significant loss in accuracy for the runways used in this investigation. Fortunately, the runways used in this investigation had no grade and in each case the maximum deflections were less than 0.5 foot above or below a mean elevation. This rather low variation of vertical deflection simplified the scaling and allowed the resolution desired to be easily obtained. If the presence of grades or long wave length deflections cause the scaling to be such that the resolution of the smaller wave length bumps begins to deteriorate, mean grade lines having wave lengths sufficiently long to cause negligible airplane response may be established and easily removed in the digital-to-analog conversion process.

Location of Rough Sections of Runway

A visual inspection of the runway profile data will give some information relative to the location of rough areas or "bumps." It should be kept in mind when viewing a single profile such as shown in figure 3, that the pitch and translation inputs result from the runway elevation at the nose gear $z_{R,n}$ and the elevation 50 feet rearward at the main gear $z_{R,m}$. The visual estimation of the bumps which might have sufficient amplitude and wave lengths to cause too much airplane response is made more difficult by the persistence of airplane response motions for some time after a particular input. The persistence of motion results in the airplane response to one section of runway being very much dependent on the preceding sections of the runway. The runway distance for which the airplane motion persists after a disturbance is dependent on the airplane frequency-response characteristics and the speed along the runway. It then appears obvious that airplane-response time histories must be used to study airplane response to a specific runway input since the airplane response to one area of the runway is dependent on the preceding time history.

The time histories of the simulated airplane response to runway A at speeds of 90 and 150 knots are shown in figure 3. After studying the time histories for the range of speeds from 50 to 160 knots, several distinct runway irregularities were identified as being the primary cause of the various peak accelerations at the pilot's compartment. These areas are shown in figure 3 and identified as bump numbers 1 to 11. For each constant speed run in each direction, the maximum accelerations at the pilot's compartment due to various bumps were plotted as a function of the location of the bump along the runway. (See fig. 4(a).) Only the accelerations of $\pm0.5g$ or greater were considered and where more than one oscillation of this magnitude appeared to result from a specific bump, only the highest value was plotted. Similarly, the maximum airplane pitch response θ is given in figure 4(b). In general, the same bumps cause the greatest response in both pitch and vertical acceleration. However, the acceleration response is greatest at the high speeds whereas the pitch response is greatest at the lower speeds. The pitch data are presented here, but it is felt that the vertical-acceleration response is more significant and throughout this paper the discussion will be primarily concerned with the vertical acceleration at the pilot's compartment.

After consideration of the data given in figure 4 and the time-history data, the areas needing repair were determined.

Method of Runway Repair

Because of the nature of this preliminary study, the final "runway repairs" were dictated by the ease with which the runway analog could be electrically limited in the positive or negative direction. Each repair was made by means of a diode limiting circuit which prohibited the runway voltage from exceeding a given positive or negative level; thus all the repaired surfaces were horizontal and no repairs which would call for a sloped line were attempted. As a first trial, the amount of repair at each bump was based on a judgment of the minimum repairs necessary. At the beginning of runway A, as shown in figure 5, the nose- and main-gear runway input circuits were limited so that the positive voltage would not exceed a value which was equivalent to 0.15 foot of elevation. After both the nose and main gear had passed bump number 1, an integrator, which generated a voltage proportional to time, caused a relay to switch the nose and main-gear limiting circuits so that the runway analog voltage would not go below zero volts. After both the nose and main gear had passed bump number 2 the limit level was changed again and, in this manner, the runway repairs shown in figure 5 were accomplished. These same repairs were used for this runway in the reverse direction by controlling the same limiters in reverse timing sequence.

Results of Runway Repairs

Time histories of the airplane response at a speed of 150 knots in both directions are given in figure 6 for runway A after being repaired. The reduction of response at a speed of 150 knots is easily seen by comparing the time-history response for the unrepaired runway (fig. 3(a)) with the response for the repaired runway (fig. 6(a)). The effect of direction of travel at a speed of 150 knots can be seen by comparing figures 6(a) and 6(b). The total effect of the runway repairs may be seen by comparison of the summarized data for the unrepaired runway given in figure 4(a) with the summarized data for the repaired runway given in figure 7. The response due to all the bumps has been lowered considerably and is ±0.5g or less, everywhere except for bump 5 in both directions and one isolated case for each of bumps 6 and 2 in the reverse direction. The acceleration response due to bump 5 is given in figure 8 as a function of speed for each direction. Comparison of the repaired and unrepaired data, particularly for the reverse direction, indicates that the repairs had no appreciable effect at some speeds. The fact that the level of accelerations due to bump 5 was not reduced to ±0.5g or less is easily understood when the repaired section is compared with the unrepaired section. (See fig. 8.) The only simple means of simulating repairs to this area while keeping the length of the repaired section at a reasonably small value was to level the top of the bumps as shown. Intuitively, the repairs accomplished in this region would seem to be less desirable than some other form of repair which might be just as easily accomplished physically as the method used here but could not be simulated easily.

Bump 8 appeared to be of rather short wave length and would be assumed to have a low harmonic content in the frequency range of interest for the simulated airplanes. However, the response caused by this bump (fig. 9) was rather large at various speeds through the speed range. When considered in relation to the sections of runway preceding bump number 8, the airplane motions in some instances are such that bump number 8 reinforces and thus amplifies the motion already in progress. Removal of this bump as shown here limited the maximum response values below the desired ±0.5g level. This particular bump was the result of previous runway repair efforts, that is, the profile shown here was the contour of a macadam patch which had been placed in this area. The condition of the surface which necessitated the macadam patch is not known, but it would appear that this patch has produced an undesirable result. It should be noted that the results for the reverse direction include the effects of repairs at both bump number 8 and the bump near the 3,000- and 3,100-foot stations.

Effects of Varying the Amount of Runway Repairs

The effect of variations in repairs were studied in some detail on bumps 1 and 2 and the data are presented in figure 10. For the forward direction, the response to bump 1 for the unrepaired runway increases with increasing speed up to a value of 1.05g at a speed of 160 knots. By repairing bump 1 for a distance of 100 feet and doing no repairs on bump 2 [(a) repairs], the response due to bump 1 and also that due to bump 2 was reduced to a suitable level except for a value of 0.52g due to bump 2 at 130 knots. The spacing between these bumps is such that at a speed of 130 knots the decaying airplane motion due to bump 1 is reinforced by bump 2 so that the resulting oscillation due to bump 2 is higher than for the other speeds.

For the more extensive repairs identified as "(b) repairs," bump 1 was repaired for a length of 140 feet as compared with 100 feet for the previous repair and bump 2 has been repaired by filling the small depressions near the 600- and 700-foot stations. The further reduction in response due to these repairs can be seen in figure 10 (diamond symbols).

The response reduction at bump 2 resulting from the more extensive repairs is probably due more to the greater reduction at bump 1 than to beneficial effects from the repairs at bump 2. For the reverse direction where bump 2 is not influenced by bump 1, the response due to bump 2 when unrepaired is above 0.5g only at 150 and 160 knots. With the extensive repairs (diamond symbols) the response is reduced but is still slightly high at 160 knots.

The less extensive repairs, that is, no repairs at bump 2, were tested at 160 knots and the reduction in response was almost as great as that when bump 2 was repaired. Thus, it would appear that, for the reverse direction, the reduction in response due to bump 2 is due in a large part to the reduction from the repairs at bump 3. Therefore, it would seem that the 125 feet of repairs at bump 2 are not very effective and, furthermore, it would be possible to eliminate the need for repairs at bump 2 by proper repairs at bumps 1 and 3.

Before actually making runway repairs, a complete study should be made of each significant bump and its relation to other bumps. The magnitude of repairs should be investigated until the minimum repairs are determined which will just get the response level down to the maximum values allowable. Several tracks, each displaced laterally across the runway, should be investigated in order to map the area needing repair.

Comparison of Runways B and C

There has been some question raised lately regarding the interpretation of runway data in the form of power spectral densities. Based on pilots' reports, runway B was thought to be excessively rough whereas runway C was thought to be satisfactory by comparison. The validity of this comparison is uncertain since runway B was a 10,000-foot runway which had been in service for some time while runway C was a 7,000-foot runway recently placed in service for turbojet transports. This fact could account for the lack of complaints as well as the fact that operation from the shorter runway would involve lighter gross weights and lower speeds. The power spectra of both runways B and C appear, within the accuracy of computation, to be about the same for the wave lengths of interest (60 to 200 feet) for the simulated airplane. (See fig. 11.) It seemed to be of interest to examine the time histories of the response of the same airplane operating on each runway. Figure 12 shows the runway profiles and corresponding response time histories of the vertical acceleration at the pilot's compartment for runways B and C with an airplane speed of 150 knots. The predominant wave length of the response in each case is about 200 feet. The power spectral data indicate about the same value for either runway for this wave length; however, the time histories indicate very obviously a lower response for runway C. These data are summarized in figure 13 for speeds between 50 and 160 knots. Each acceleration peak greater than $\pm 0.5g$ was plotted at the runway station where it occurred; thus, both the number of points and the magnitudes of the points are a measure of the relative roughness of the two runways. The response due to runway B appears to be definitely greater than that due to runway C for speeds of 160, 150, 140, 100, 70, 60, and 50 knots; the response is about the same for each runway at speeds of 130, 120, and 90 knots. The response due to runway C is greater for speeds of 110 and 80 knots. A comparison of time histories shows that runway B was rougher in most cases than runway C whereas the power spectra indicate about the same roughness. Power spectra based on whole runway averages do not appear to be suitable for the type of problem considered in this investigation where specific areas are of interest.

CONCLUSIONS

A study of several rough runway profiles has been made by utilizing the time histories of the response of a simplified airplane while taxiing on the runway. The simplified airplane was simulated on an electronic analog computer and the runway analog input was in the form of an FM magnetic tape. The results of this study have indicated the following conclusions:

1. The digital-to-analog conversion from runway survey data to an FM magnetic tape is feasible and provides a convenient means of generating a repeatable analog representation of a given runway profile.

2. An evaluation of the simulated airplane response to runway roughness by means of time histories appears to be an appropriate means of locating relatively rough areas of a given runway.

3. A detailed study of each rough area may be made by this method to determine the minimum repairs necessary to reduce the airplane response to a suitable level.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., July 13, 1962.

APPENDIX

GENERATION OF THE ANALOG RUNWAY PROFILES

Symbols

The symbols used in this appendix are defined as follows:

$f(t)$	function of time
$f'(t)$	reconstituted function of time (after sampling and conversion)
$f(X)$	function of the arbitrary argument X
$G(\omega)$	transfer function of amplifier
$G_C(\omega)$	transfer function of digital-to-analog converter
$G_I(\omega)$	transfer function of interpolated values
j	unit imaginary vector, $j^2 = -1$
m,n	positive integers
$S(t)$	zero-aperture sampling function
t	time, sec
T_H	data holding time of digital-to-analog converter, sec
T_S	effective time between survey points, sec
V	taxiing speed, ft/sec
X_n	arbitrary argument
λ_S	distance between survey points, ft
τ	time constant, sec
ω	frequency, radians/sec
ω_S	effective frequency of survey samples, radians/sec

Digital-Analog Conversion Technique

The digital and analog data-processing techniques that were used in the generation and recording of the analog runway-profile time histories are presented in detail in this appendix. Commercially available equipment, or equipment of similar type, was used for all the data-processing steps.

The employment of digital computing equipment permitted the use of a numerical interpolation technique which has distinct advantages over analog data-smoothing methods. In addition, the insertion of a fixed-distance lag between the occurrence of a profile stimulus at the nose gear and at the main landing gear was readily accomplished. The digital-to-analog conversion method described did not require the use of curve-following equipment. The analog signal did not, therefore, suffer from the loss of accuracy inherent in the manual curve fairing that is required when such equipment is used.

The amplitude accuracy of the reproduced analog signals was of the same order as the initial survey. The dynamic fidelity of the analog signals was consistent with the requirements of the simulation problem. For simulation problems requiring greater dynamic fidelity, a more complex numerical interpolation technique can be substituted for the one described.

Operations and Equipment

The steps below were followed in the conversion of the tabulated profile data to analog signals.

(1) A punched card deck was prepared which consisted of:

(a) Scaling information (maximum, minimum, and initial elevations, and the average of the maximum and minimum elevations)

(b) Survey data, one elevation per card. The initial and final elevations were each repeated 400 times to provide regions of constant elevation, faired into the runway; thereby spurious terminal disturbances in the landing-gear response simulation when the analog computer was started and stopped were prevented.

(2) The cards were processed through an IBM 7070 data processing system which:

(a) Subtracted the average of the maximum and minimum elevations from each of the items on the scaling information and recorded the differences on a digital magnetic tape. These differences provided zero, full-scale, and initial condition calibrations for scaling the analog computer inputs. Each value has recorded 800 times to insure that the resulting calibration signals would be of adequate duration.

(b) Biased all survey data by the average of the maximum and minimum elevations; computed interpolated values between survey stations; lagged the elevation data by 50 points and wrote a tape record for each point containing the biased elevation for that point (surveyed or interpolated) and for the lagged point. The lagging provided simultaneous elevations for the nose gear and the main gear.

(3) The digital magnetic tape was played back and the digital data were converted to analog signals as explained below. The analog signals were recorded on magnetic tape as a wide-deviation FM signal.

(4) The runway analog was then provided by playback of the analog tape and demodulation of the FM signals with the discriminator outputs directly connected to the analog computer.

(5) To provide data for the simulation of taxiing in the reverse direction, the elevation cards were reverse sorted, the final elevation substituted for the initial in the scaling information deck, and the entire process was repeated.

The original runway profile surveys had been made at 2-foot intervals. The method of the digital interpolation and its effect on the dynamic fidelity of the analog reproductions of the profiles are described in detail in a subsequent section.

The control and conversion unit of a Beckman Instruments, Inc. digital-tape-to-plotter system was used to convert the elevation data from digital to analog form. The conversions were made at a fixed rate of 22.2 tape blocks per second. Since elevation interpolations had been made at 1-foot intervals, the analog signals were equivalent to the nose and main gear elevation inputs at a taxiing speed of 22.2 feet per second.

The analog signals were obtained from the buffer operational amplifiers which normally drive the abscissa and ordinate inputs of a plotboard. The closed-loop frequency response of these amplifiers is:

$$G(\omega) = \frac{1}{1 + j\tau\omega}$$

The time constant τ has a minimum value of 100 milliseconds when the equipment is used for normal plotting operations. To make the equipment response suitable for the profile-data-conversion operation, the time constant was reduced to 30 microseconds.

A wide-band FM tape recorder was used to record the profile analogs and the output of a block counter which emitted a pulse at 100 block intervals. The data were recorded at a tape speed of 30 inches per second. Tape speeds of both 30 and 15 inches per second were used when the data were reproduced in order to provide greater flexibility in scaling the analog computer to simulate a wide range of taxiing speeds. The FM discriminators that were used were not integral parts of the FM tape system. The discriminators used had a greater full-scale output range than the type usually furnished with tape systems (±30 volt instead of ±1 volt) and in this sense were more suitable for use with the analog computer. The output sections of the discriminators used 50-cycle-per-second low-pass filters of the "constant-amplitude" type.

Accuracy and Dynamic Fidelity

The analog reproductions of the previously mentioned computer scaling records were also used to verify the accuracy of the analog reproduction of the data. Constant elevations were reproduced with a total error of less than 0.01 foot.

The dynamic fidelity of the reproduced profiles was determined primarily by factors inherent in the sampling and data conversion process. Digital interpolation of values between the original survey points offered some improvement over the fidelity that could be obtained with the basic process.

To determine the dynamic fidelity, it is useful to consider the runway profiles as equivalent time functions, where time is equal to distance along the runway divided by a given taxiing velocity V. The surveyed elevations, measured at intervals $\lambda_S = 2$ feet, are equivalent to the results of sampling the time functions with a zero-aperture sampler which has a sampling frequency $\omega_S = 2\pi\frac{V}{\lambda_S}$. The frequency spectrum of the resulting sample pulses contains, in addition to the spectrum of the runway, images of this spectrum about each of the frequencies $m\omega_S$, where m is a positive integer.

The digital-to-analog converter extrapolates the data between samples by holding the data constant at the value of the last pulse, as shown in figure 14(a). The process, zero-order data holding, is equivalent to passing the sampled data pulses through a low-pass filter with a transfer function:

$$G_C(\omega) = \frac{T_H}{T_S} \frac{\sin \pi \frac{T_H \omega}{T_S \omega_S}}{\pi \frac{T_H \omega}{T_S \omega_S}} e^{-j\pi \frac{T_H \omega}{T_S \omega_S}}$$

Ideally, T_H is equal to the period between samples T_S. In actuality, the converter is reset to zero for a finite time before going to the next data level. From the transfer function it is evident that (a) the response in the region of the data frequencies is not flat and (b) the attenuation at the image frequencies is not infinite. The unattenuated images are significant only to the degree that the landing-gear simulation is responsive to them. However, a more sophisticated simulation of the airplane and landing gear than that actually used would contain higher order structural response modes which could be spuriously stimulated by the image frequencies. In consideration of a possible future use of the analog-data tapes for these more extensive studies, attention was given to further attenuation of the image frequencies in the region of the fundamental sampling frequency.

The data smoothing was accomplished, when the data were still in digital form, by interpolating elevations halfway between the original survey points. The interpolation equation used was the parabolic curve fit formula:

$$f\left(X_n+\frac{\lambda S}{2}\right) = \frac{-1}{16}f\left(X_{n-1}\right) + \frac{9}{16}f\left(X_n\right) + \frac{9}{16}f\left(X_{n+1}\right) - \frac{1}{16}f\left(X_{n+2}\right)$$

The digital-to-analog conversion equipment operated alternately on a point from the original survey and then on an interpolated value, the data being held constant for a time that was approximately proportional to $T_S/2$, as shown in figure 14(b). For an exact interpolation, the result would be the same as if the sampling rate had been doubled, except that the Nyquist frequency imposed by the original sampling does not change.

The exact effect of the interpolation upon the dynamic fidelity of the analog data can be calculated by considering an equivalent process, shown in figure 15. The output signal is treated as a composite of the conversions of the original data samples and the interpolated samples.

From frequency spectra of typical runways, it can be assumed that in effect the runways studied (considered as time functions) contained no energy above the Nyquist frequency imposed by the sampling interval. In this case, the interpolated values are equal to those obtained by first passing the unsampled functions through a filter having the transfer function

$$G_I(\omega) = \left(\frac{9}{8} \cos \frac{\pi\omega}{\omega_S} - \frac{1}{8} \cos \frac{3\pi\omega}{\omega_S}\right)$$

and then sampling the result with a sampling function that lags the original by $T_S/2$.

The odd-order image spectra created by this sampling are 180° out of phase with those of the original sampling process. When the outputs of the digital-to-analog converters are summed, the odd-order images tend toward zero as $G_I(\omega)$ (in the data frequency region) approaches unity.

Figure 16 shows the dynamic response of the sampling, interpolating and converting process out to the second harmonic of the sampling frequency. The response for conversion without interpolation is also shown. The significant portions of the runway frequency spectra were in the region $0 \leqq \frac{\omega}{\omega_S} \leqq 0.2$. Interpolation improved the response in this region as well as providing almost complete attenuation of the first-order image spectrum in the region, $0.8 \leqq \frac{\omega}{\omega_S} \leqq 1.2$. The combined response of the filters at the output terminals of the digital-to-analog converter and the FM discriminator had a negligible effect upon the total response. The additional attenuation provided by these filters was calculated to be less than 0.05 percent over the region of the runway frequency spectra.

The linear phase response exhibited by either conversion method is the equivalent of a constant time delay at all frequencies. Since the data were not reproduced in real time, the delay is only hypothetical.

If required, the digital interpolation may be altered to provide interpolated elevations at closer intervals. This additional interpolation would yield both increased attenuation of the higher order images and improved response in the region $0 \leqq \frac{\omega}{\omega_S} \leqq 0.2$.

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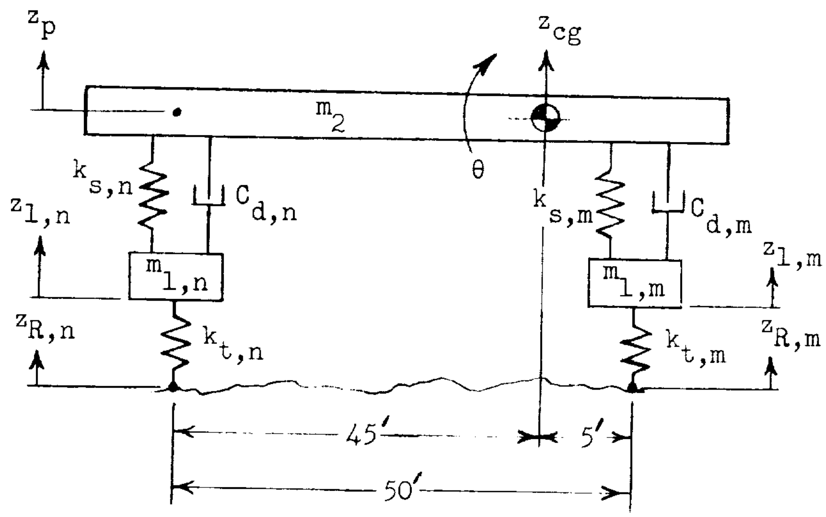


$m_T = 4350$ lb-sec^2/ft $\qquad m_2 = 4205$ lb-sec^2/ft

$m_{2,n} = 420$ lb-sec^2/ft $\qquad m_{1,n} = 15$ lb-sec^2/ft

$m_{2,m} = 3785$ lb-sec^2/ft $\qquad m_{1,m} = 130$ lb-sec^2/ft

$r = 25$ ft (radius of gyration of upper mass, m_2)

$\omega_{t,n} = \omega_{t,m} = 10$ radians/sec

$k_{t,m} = \omega_{t,m}^{\;2}\,(m_{1,m} + m_{2,m}) = 391{,}500$ lb/ft

$k_{t,n} = \omega_{t,n}^{\;2}\,(m_{1,n} + m_{2,n}) = 43{,}500$ lb/ft

$k_{t,m} = 0.5\ k_{s,m}$ and $k_{t,n} = 0.5\ k_{s,n}$

$\omega_{s,m} = \omega_{s,n} = \sqrt{\dfrac{k_{s,m}}{m_{2,m}}} = 14.4$ radians/sec

$\zeta = 0.2$

$C_{d,m} = 2\,\zeta\,(\omega_{s,m})(m_{2,m}) = 21{,}800$ lb-sec/ft

$C_{d,n} = 2\,\zeta\,(\omega_{s,n})(m_{2,n}) = 2{,}420$ lb-sec/ft

Figure 1.- The simulated airplane.

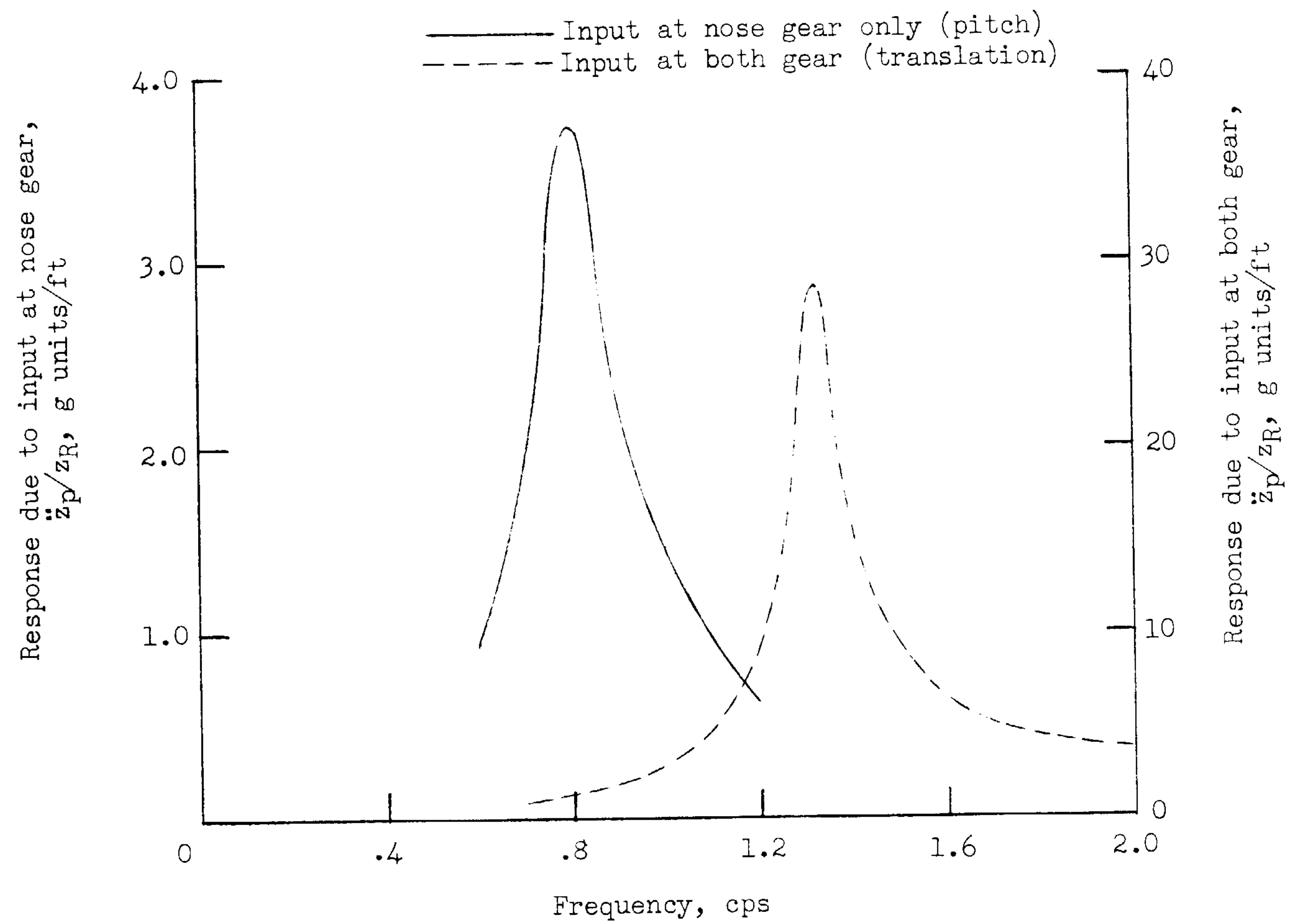
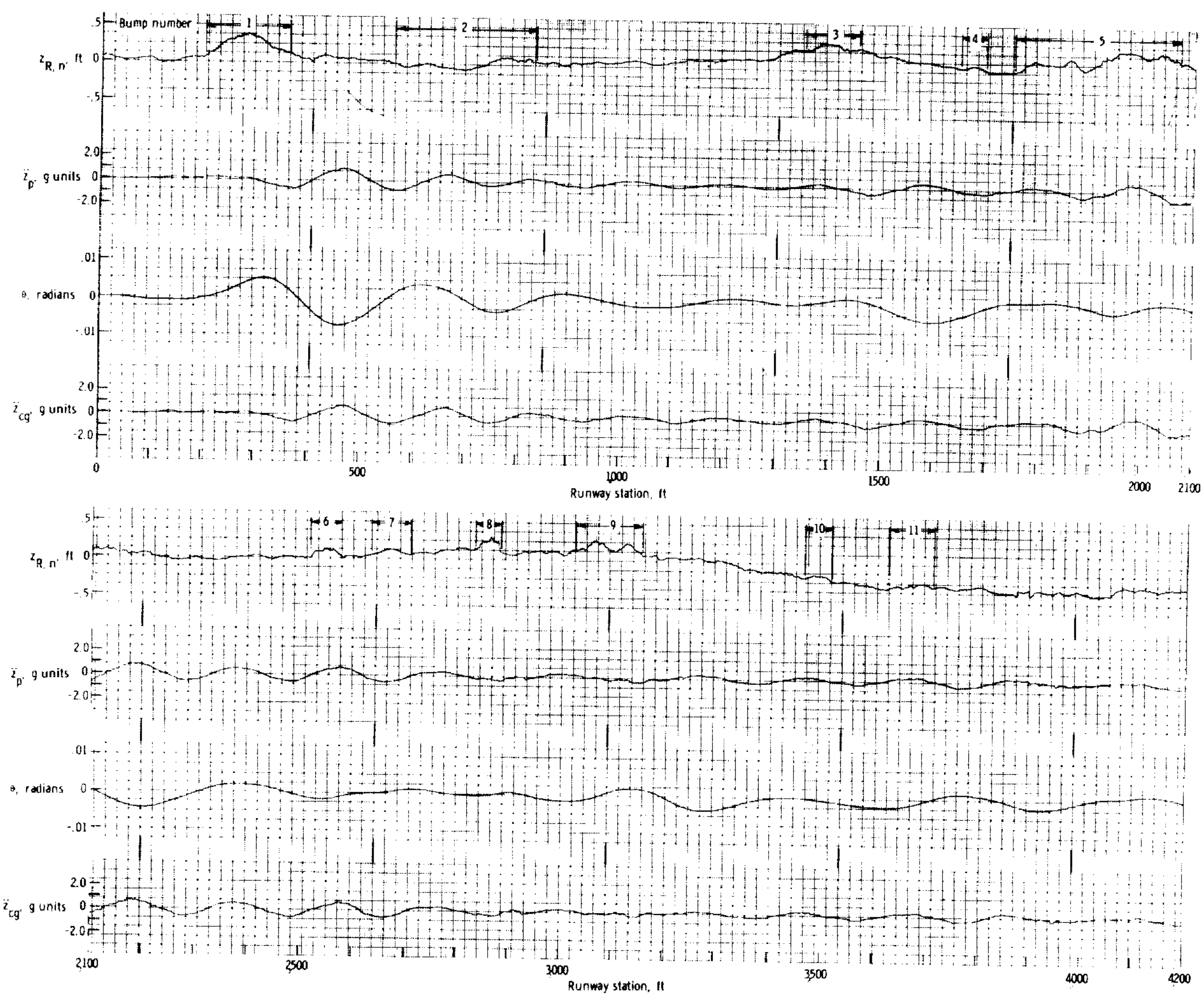
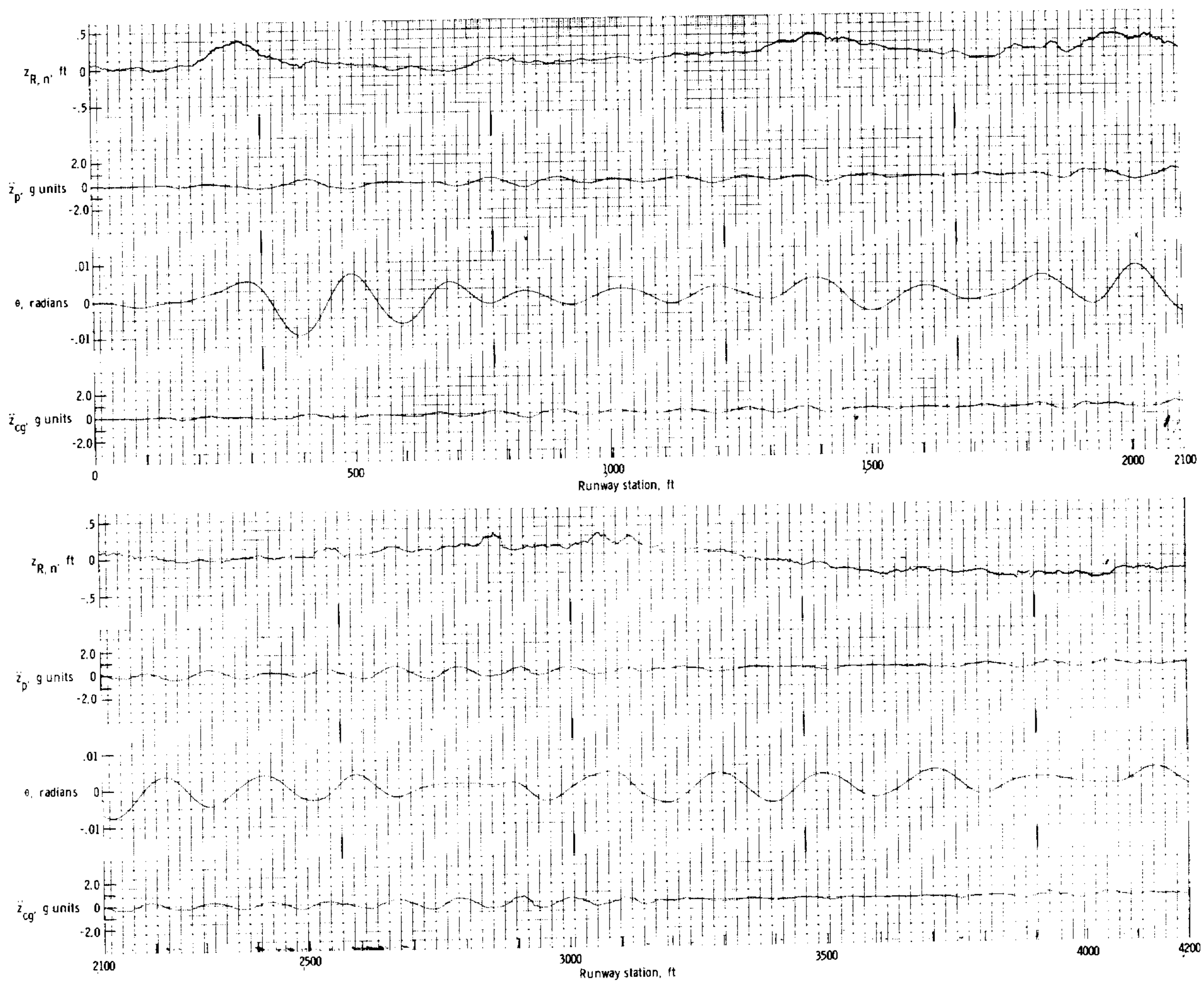


Figure 2.- Frequency response of simulated airplane.



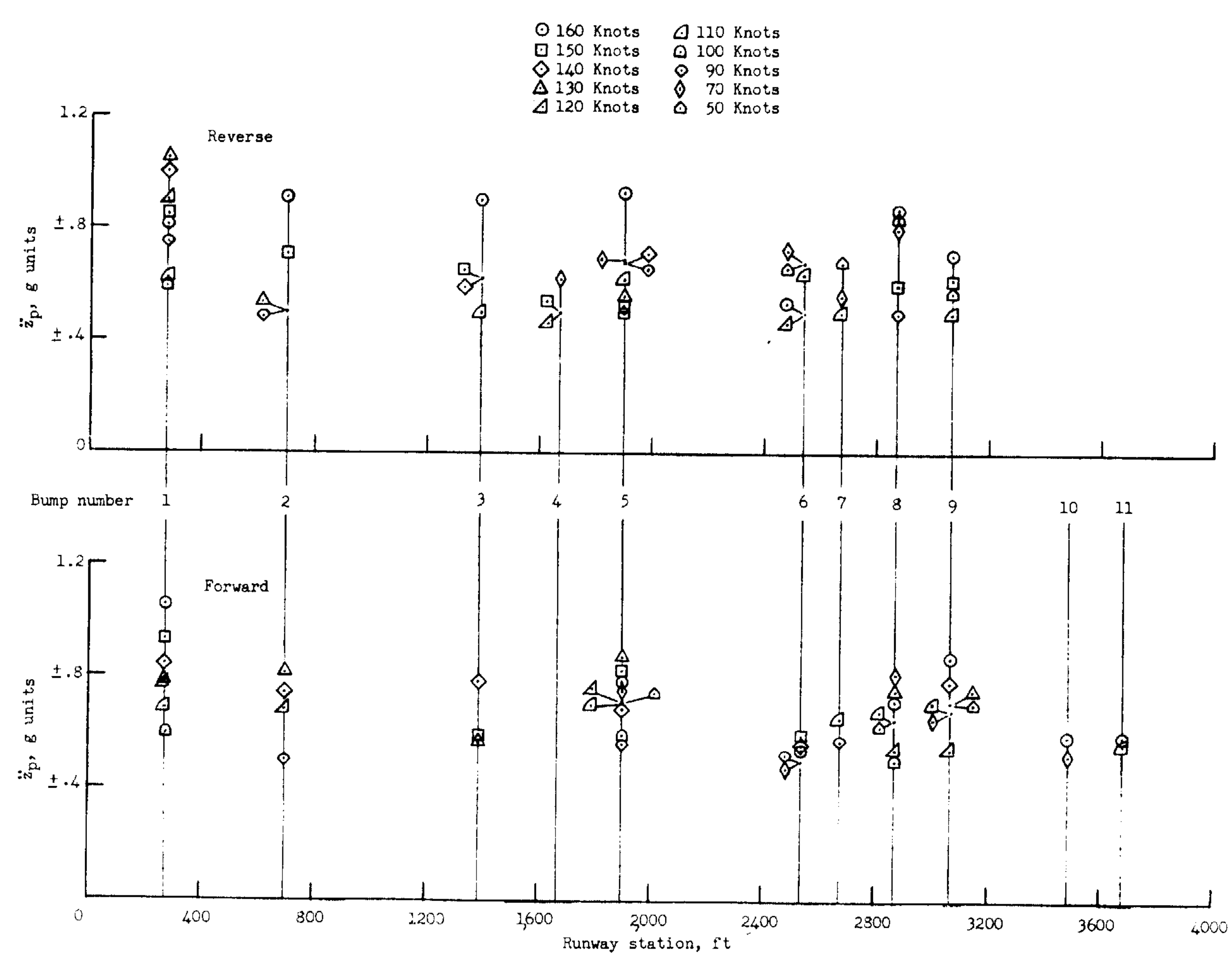
(a) 150 knots, forward direction.

Figure 3.- Time histories of airplane response while taxiing on runway A.



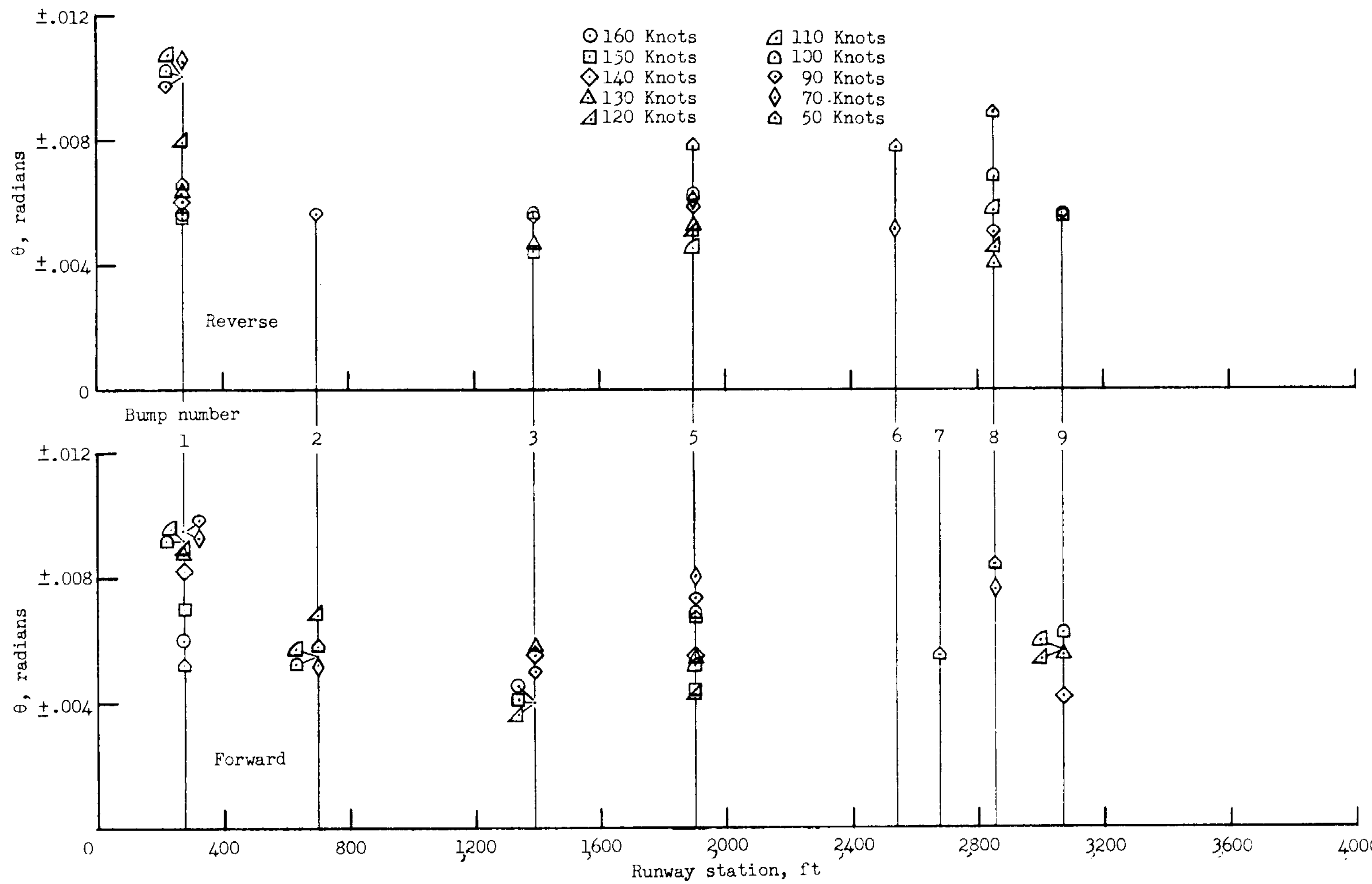
(b) 90 knots, forward direction.

Figure 3.- Concluded.



(a) Vertical acceleration at the pilot's compartment.

Figure 4.- Summary of maximum pitch angle and pilot's compartment acceleration response due to various rough areas of runway A.



(b) Fuselage pitch angle.

Figure 4.- Concluded.

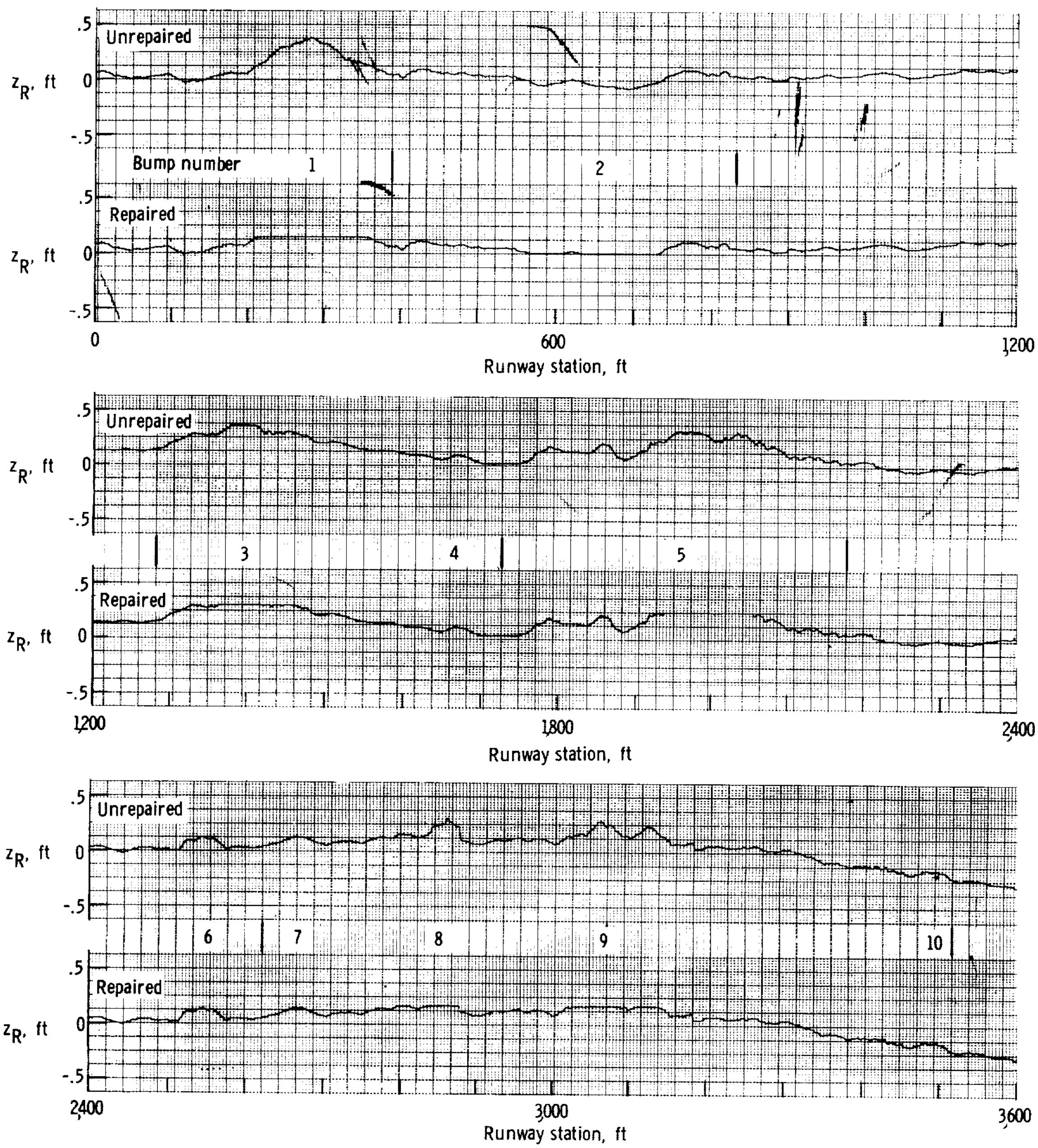
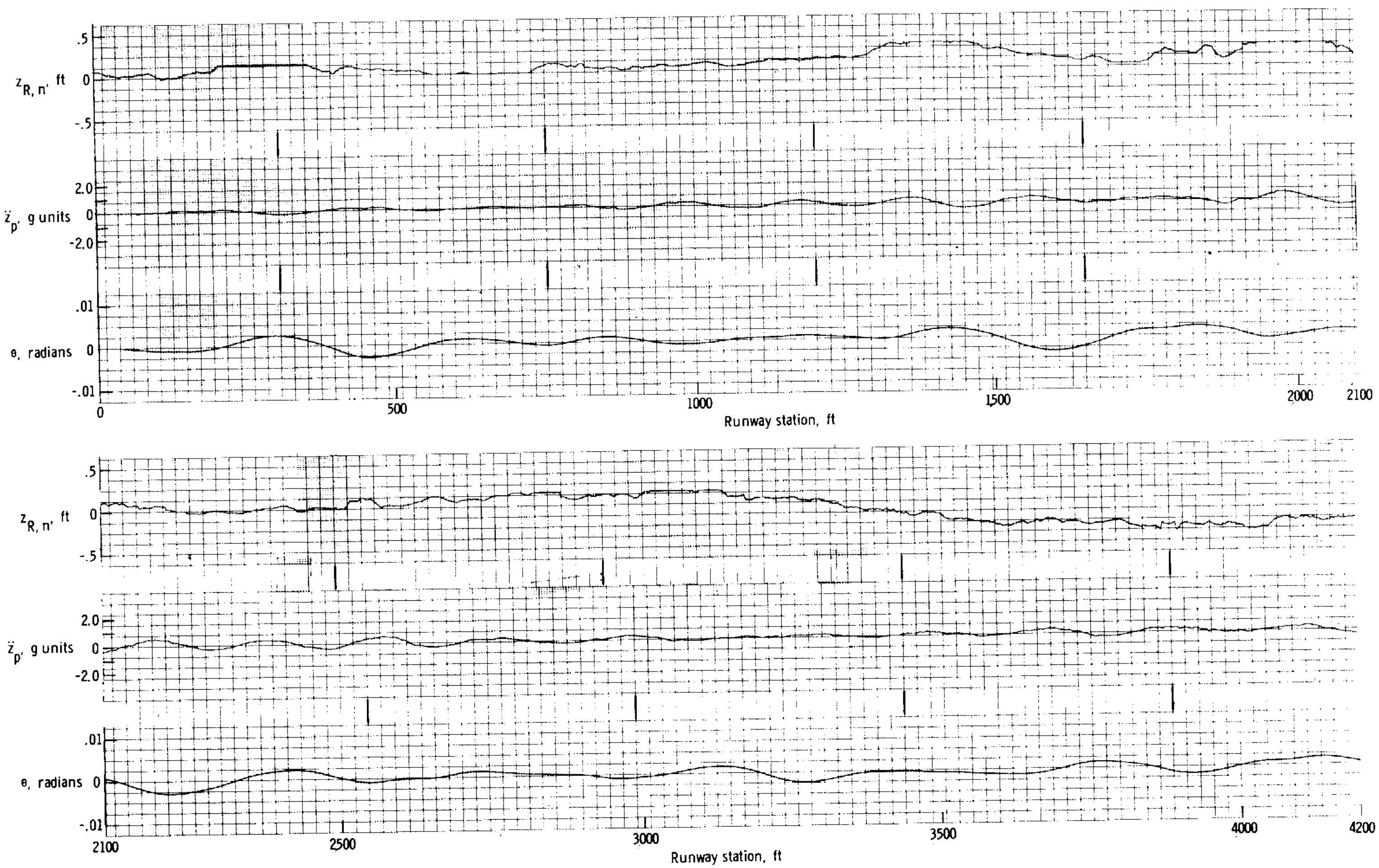
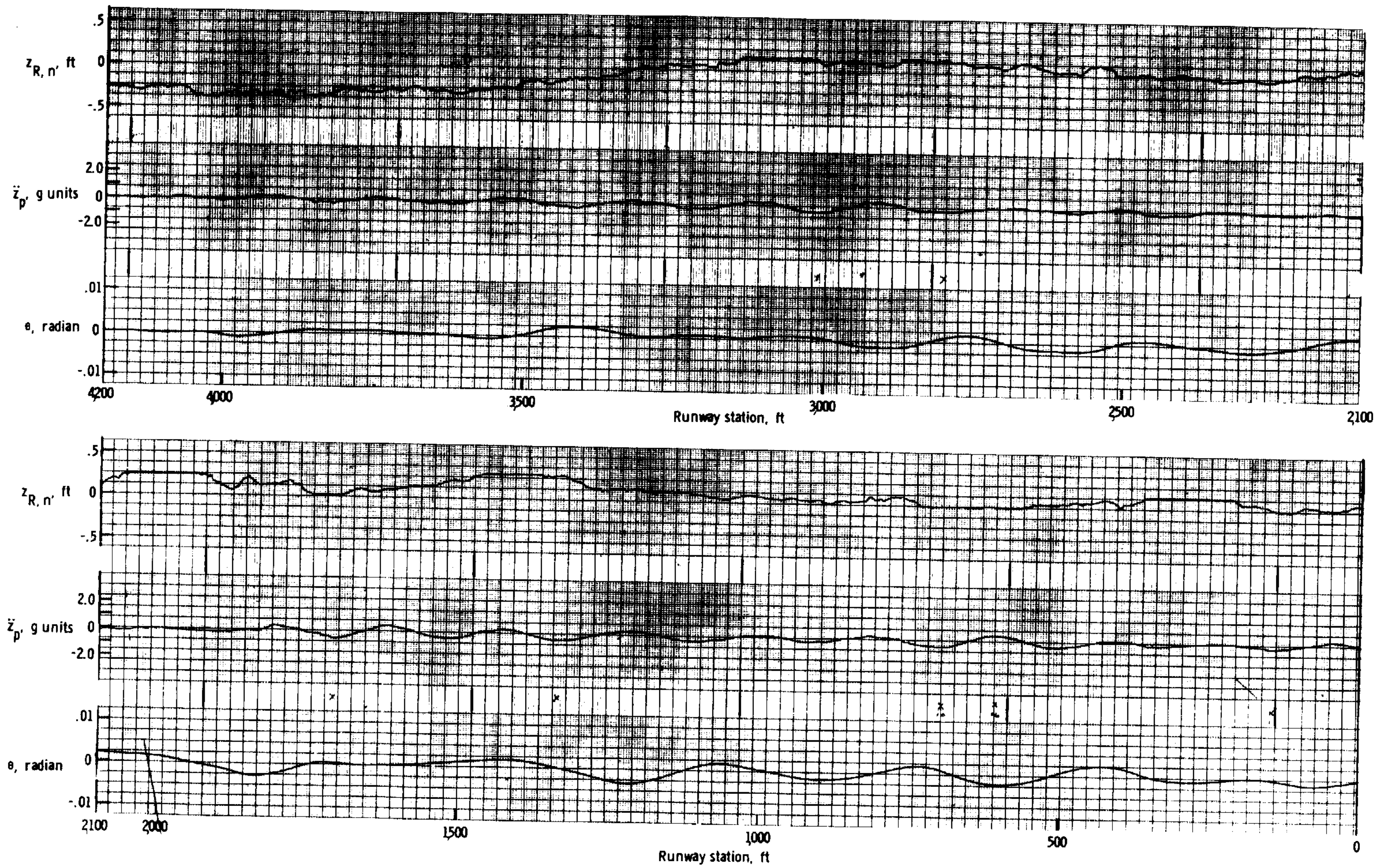


Figure 5.- A portion of the analog for runway A showing both the repaired and unrepaired runway.



(a) 150 knots, forward direction.

Figure 6.- Time histories of airplane response while taxiing on the repaired runway A.



(b) 150 knots, reverse direction.

Figure 6.- Concluded.

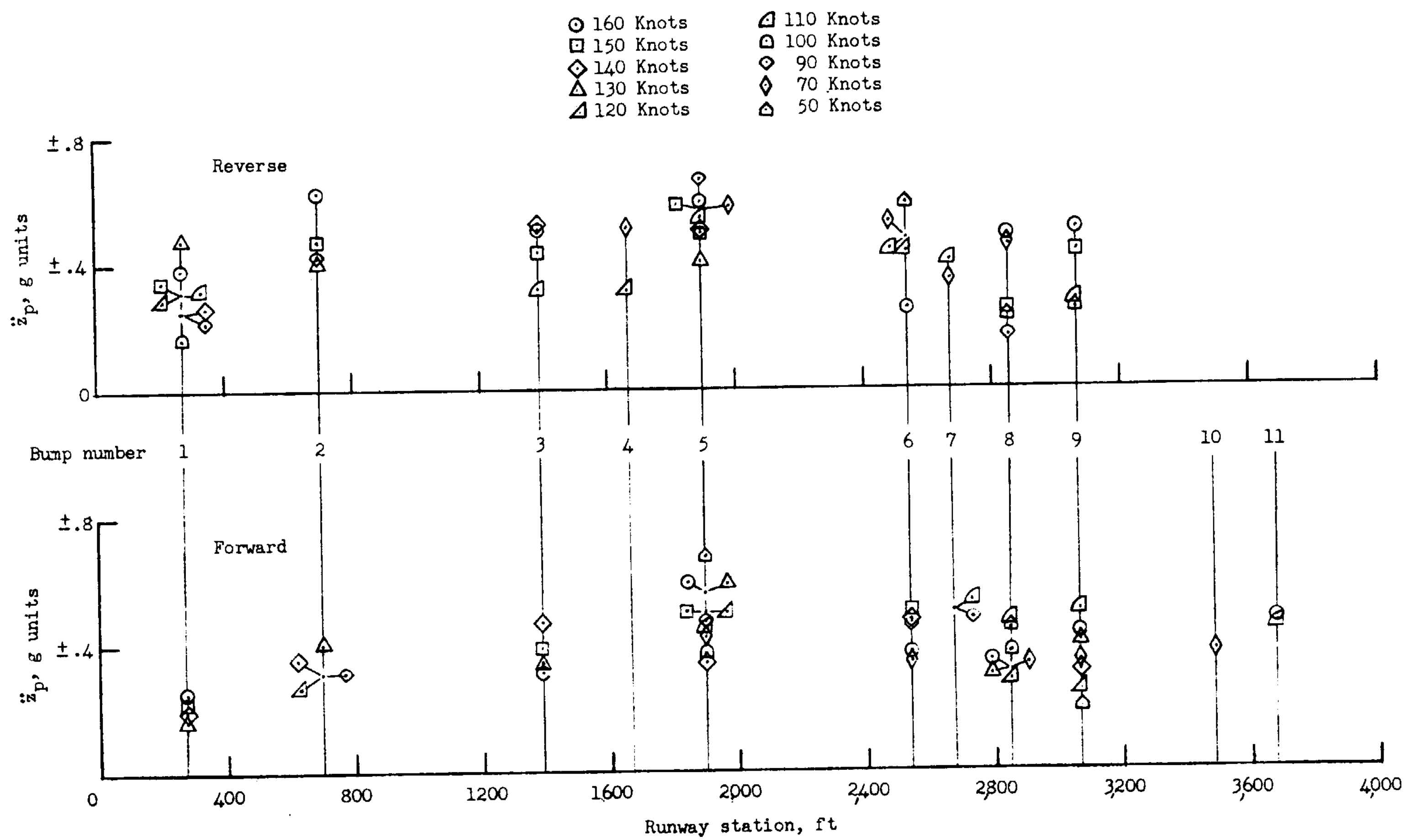


Figure 7.- Summary of maximum pilot's compartment acceleration response due to runway A after repairs.

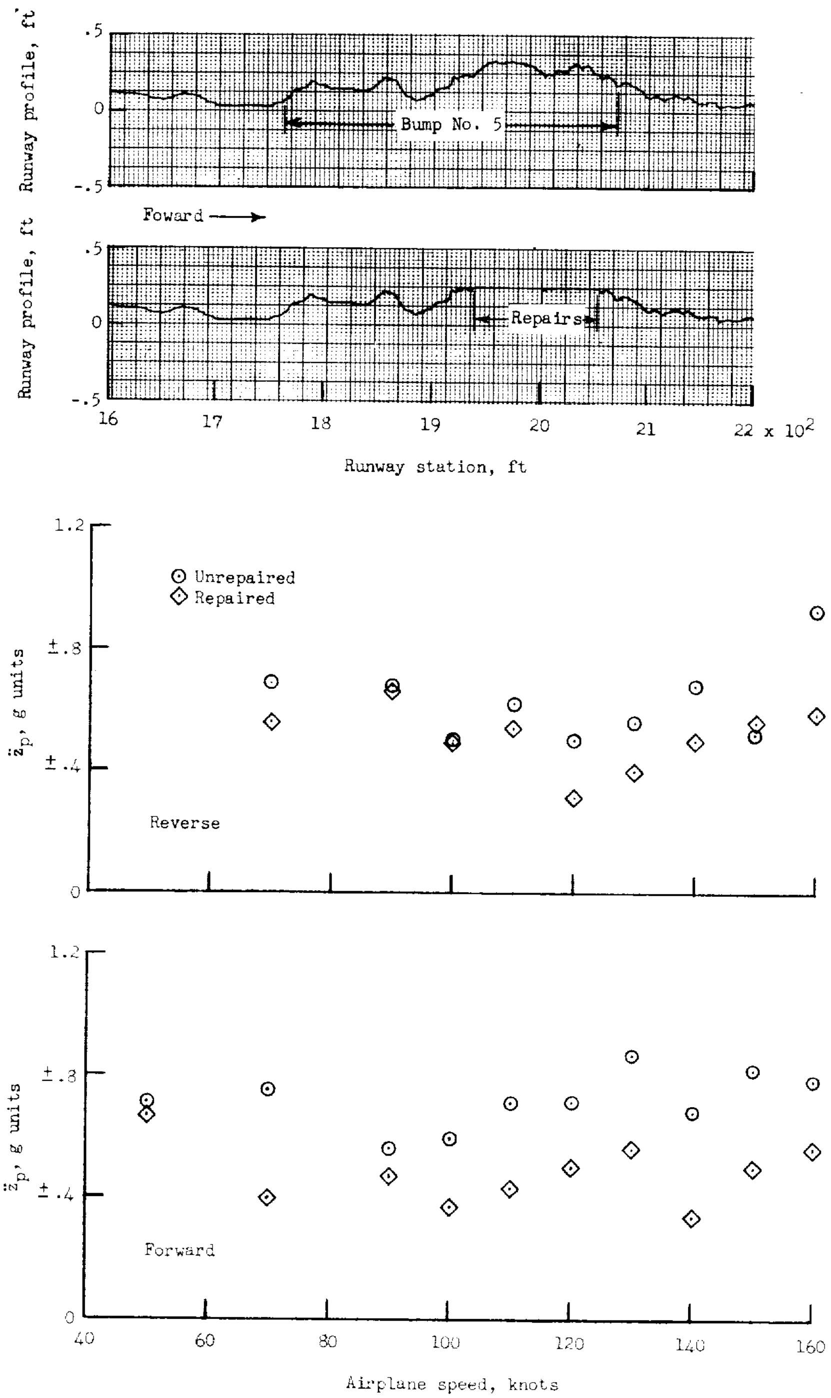


Figure 8.- Acceleration response at the pilot's compartment resulting from passing over bump number 5 at various speeds in both directions. Results are given for both the original bump and the repaired bump.

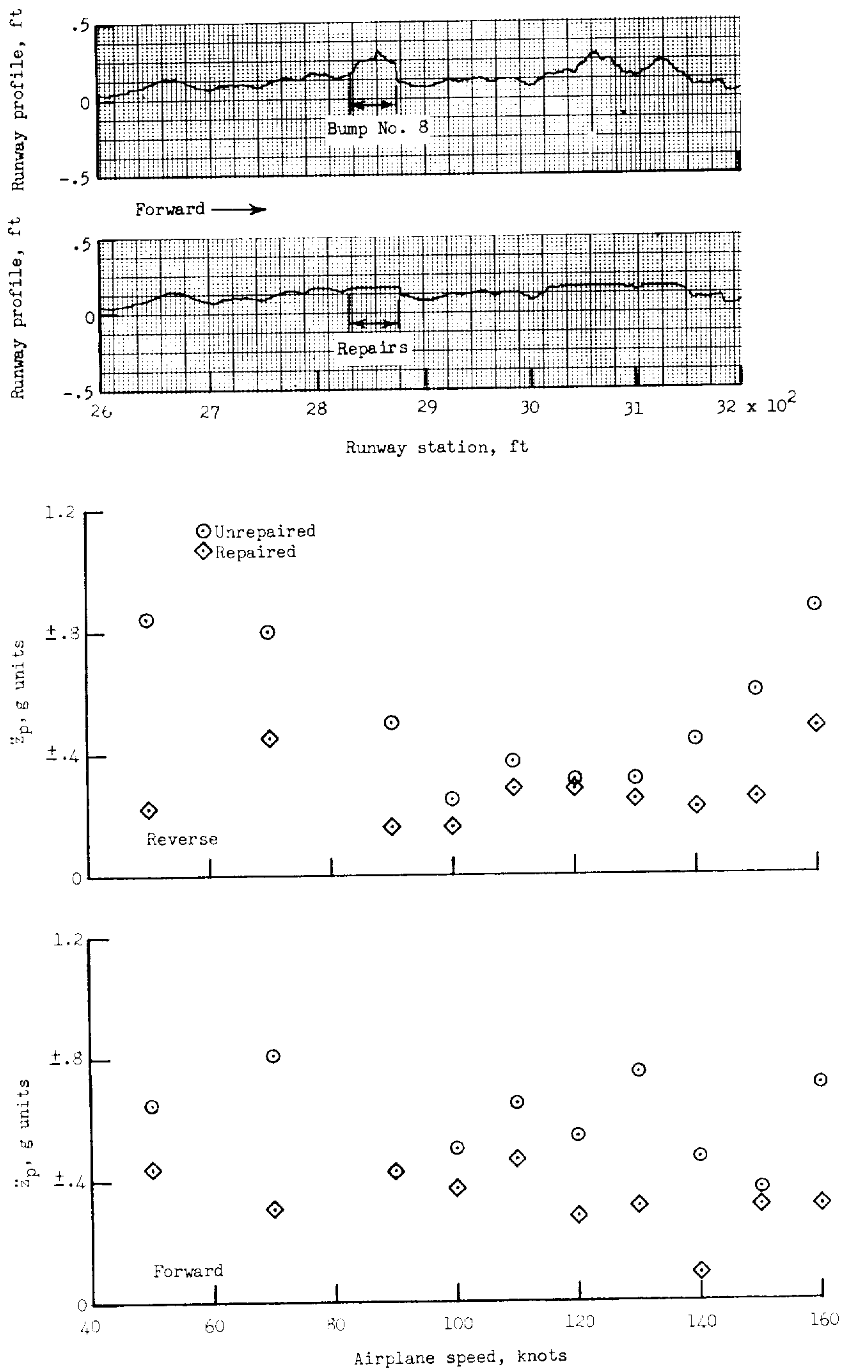


Figure 9.- Acceleration response at the pilot's compartment resulting from passing over bump number 8 at various speeds in both directions. Results are given for both the original bump and the repaired bump.

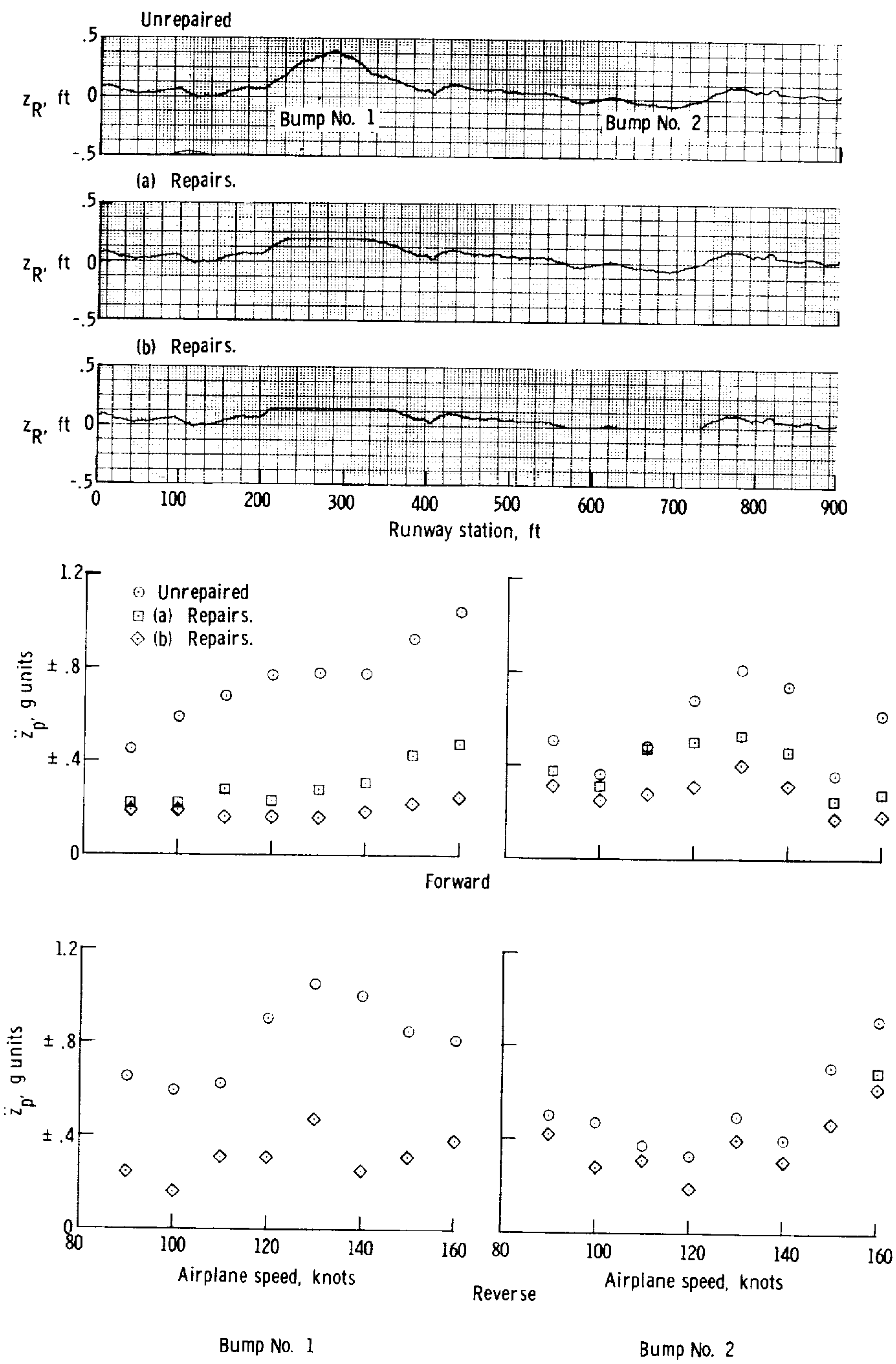


Figure 10.- The effect of various types of runway repair on the acceleration response at the pilot's compartment.

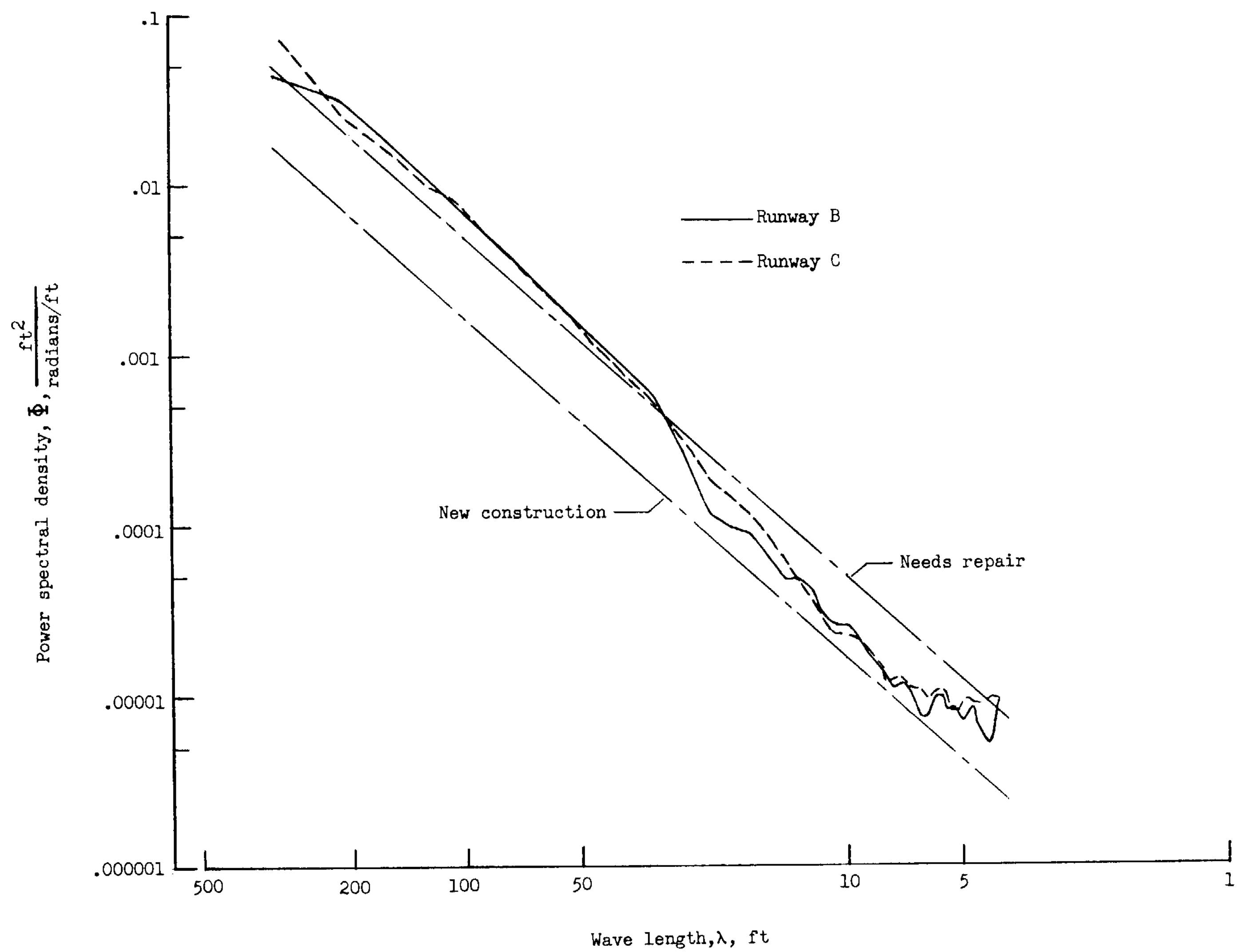


Figure 11.- Power spectral density function for runways B and C.

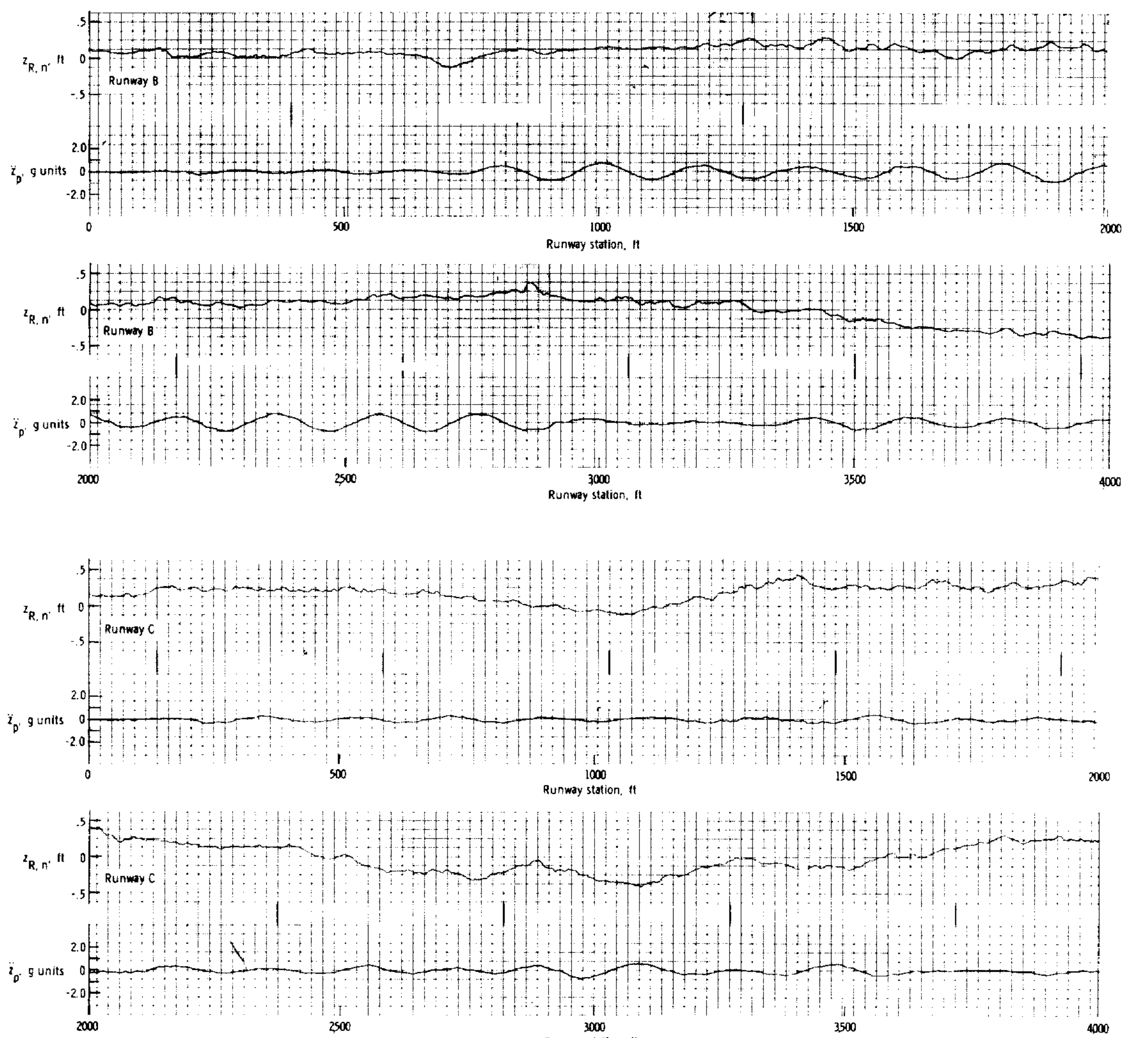


Figure 12.- Time history of airplane response while taxiing at 150 knots on runways B and C.

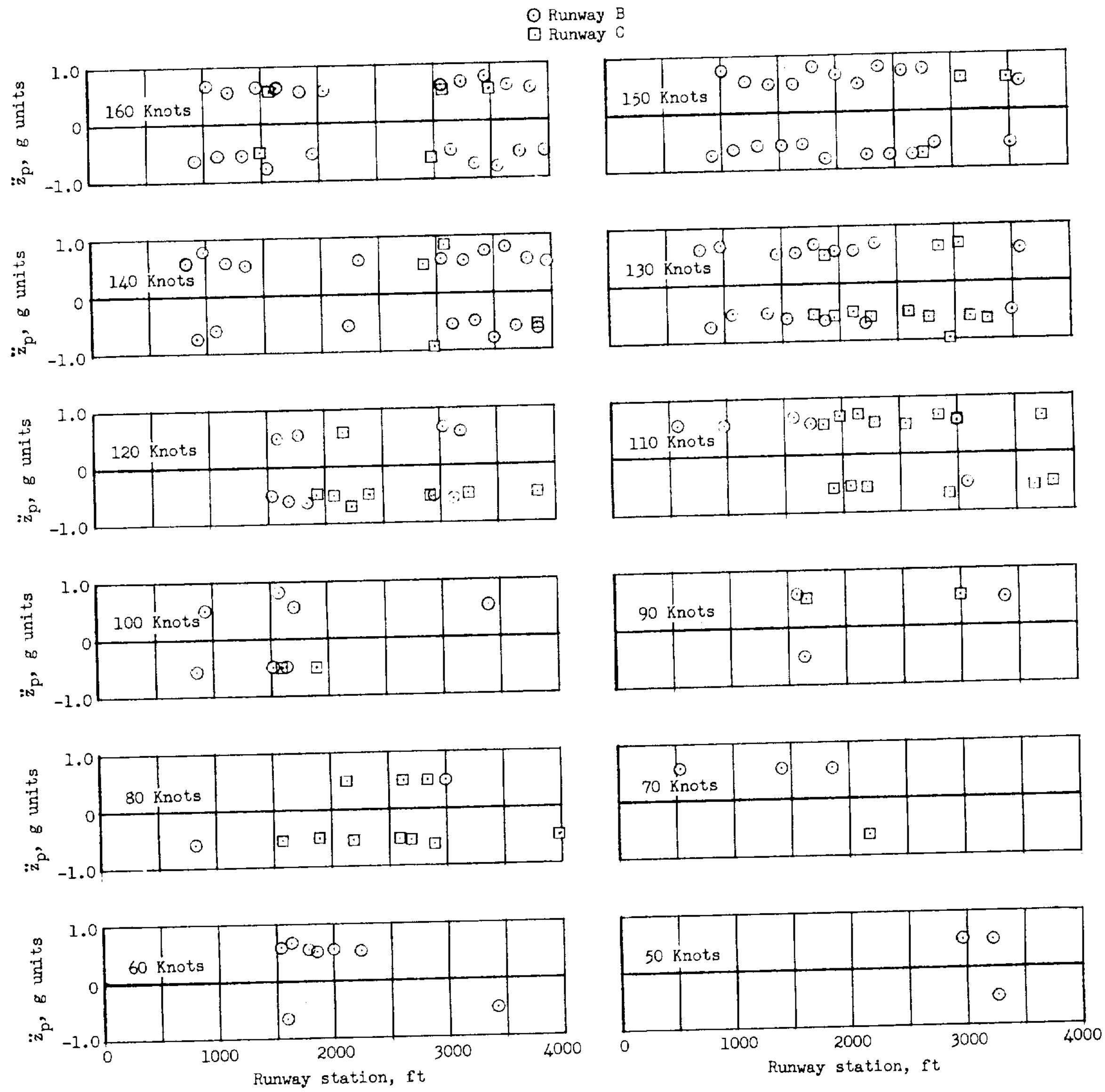
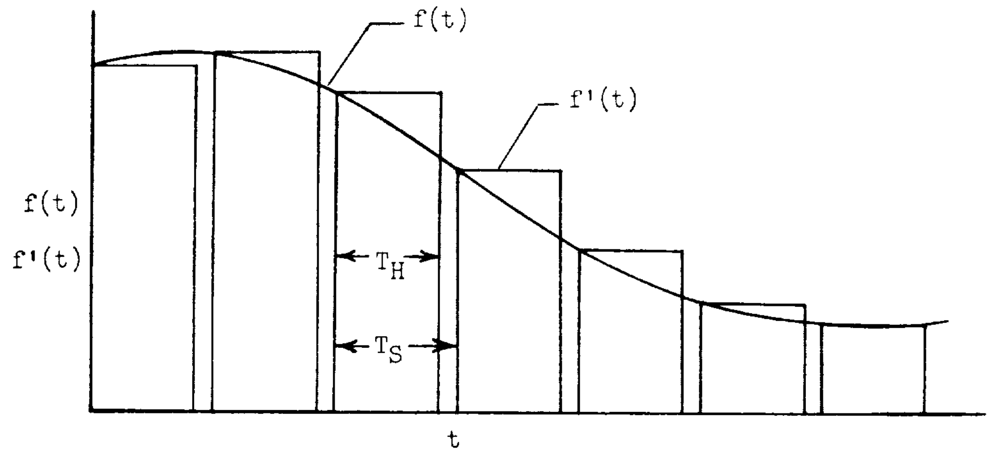
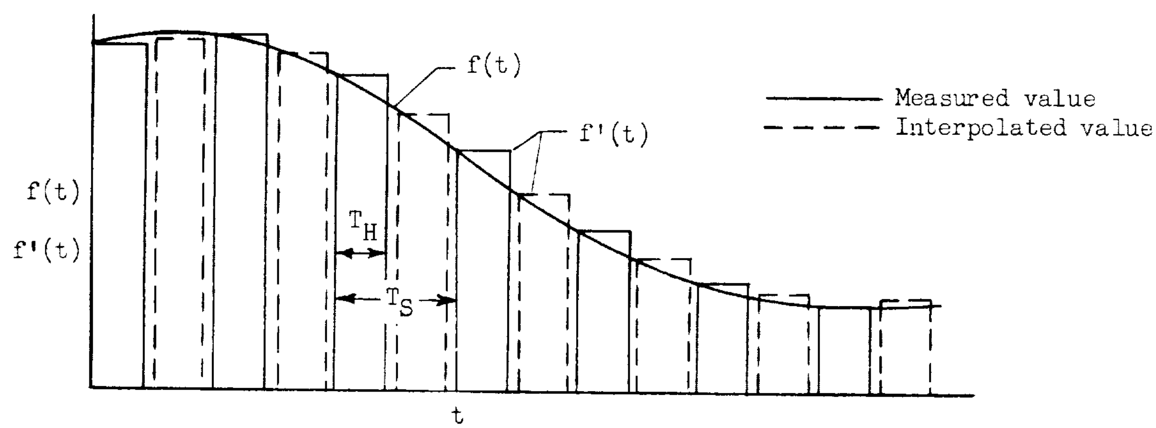


Figure 13.- Comparison of maximum accelerations at the pilot's compartment due to runway B with those due to runway C at various taxi speeds.

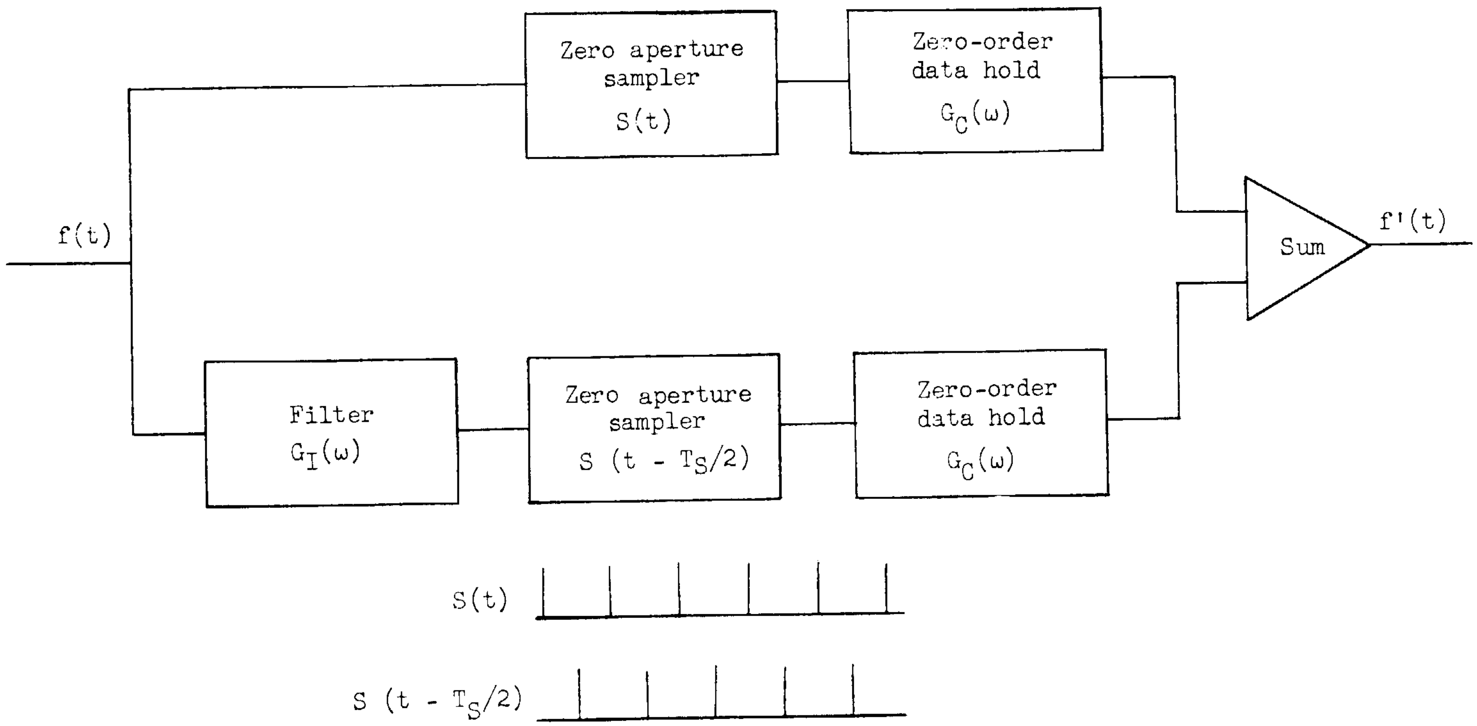


(a) Noninterpolated data.



(b) Interpolated data.

Figure 14.- Original time function and output of digital-to-analog converter.



$$G_I(\omega) = \frac{9}{8} \cos \frac{\pi\omega}{\omega_S} - \frac{1}{8} \cos \frac{3\pi\omega}{\omega_S}$$

$$G_C(\omega) = \frac{T_H}{T_S} \frac{\sin \frac{T_H}{T_S} \frac{\pi\omega}{\omega_S}}{\frac{T_H}{T_S} \frac{\pi\omega}{\omega_S}} e^{-j\pi \frac{T_H \omega}{T_S \omega_S}} \qquad \frac{T_H}{T_S} = 0.466$$

Figure 15.- Analog equivalent of interpolation.

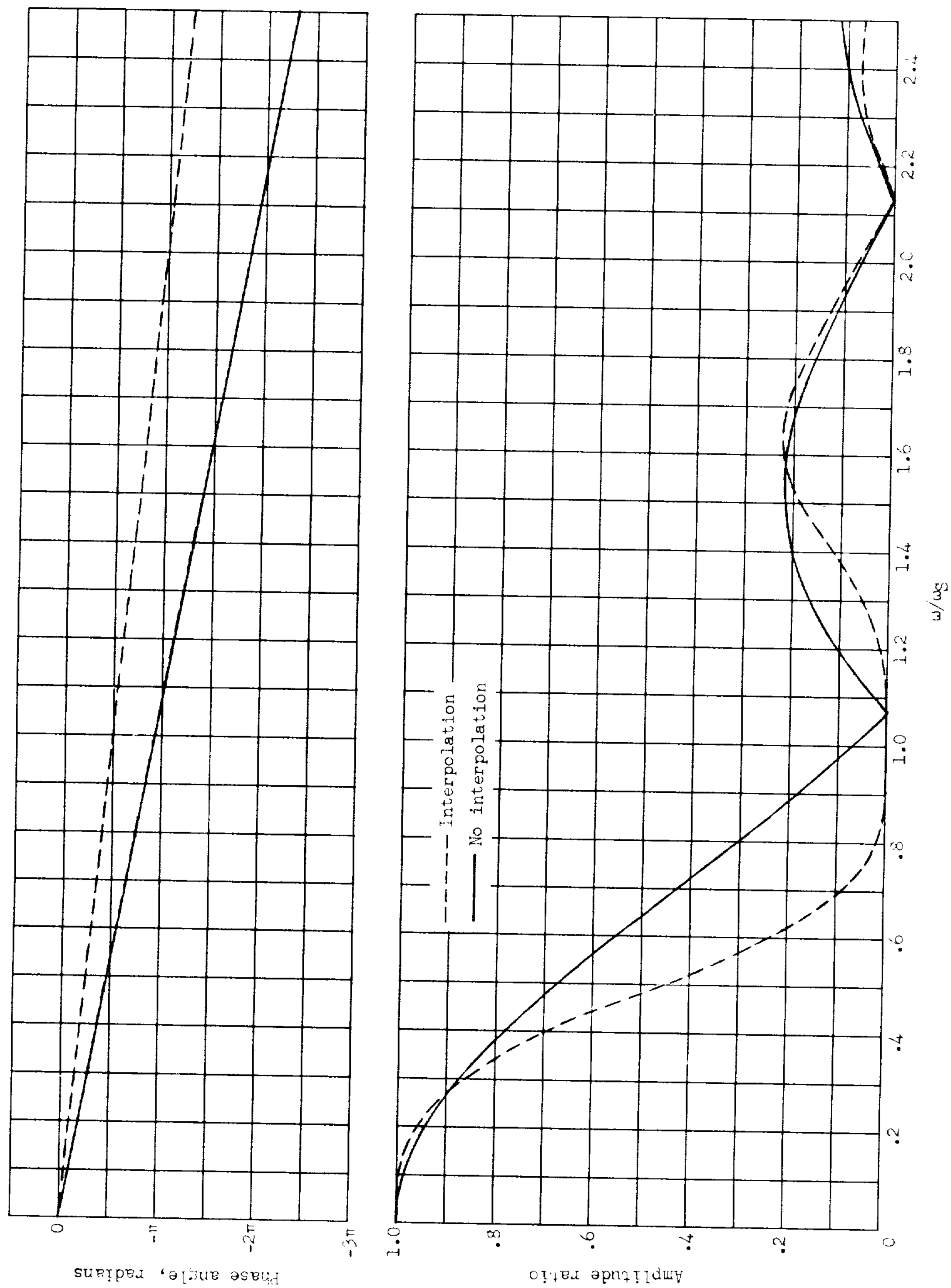


Figure 16.- Dynamic response of sampling and conversion.